EXPLODING DOTS IN TWO DIMENSIONS

By *Kiran Ananthpur Bacche*

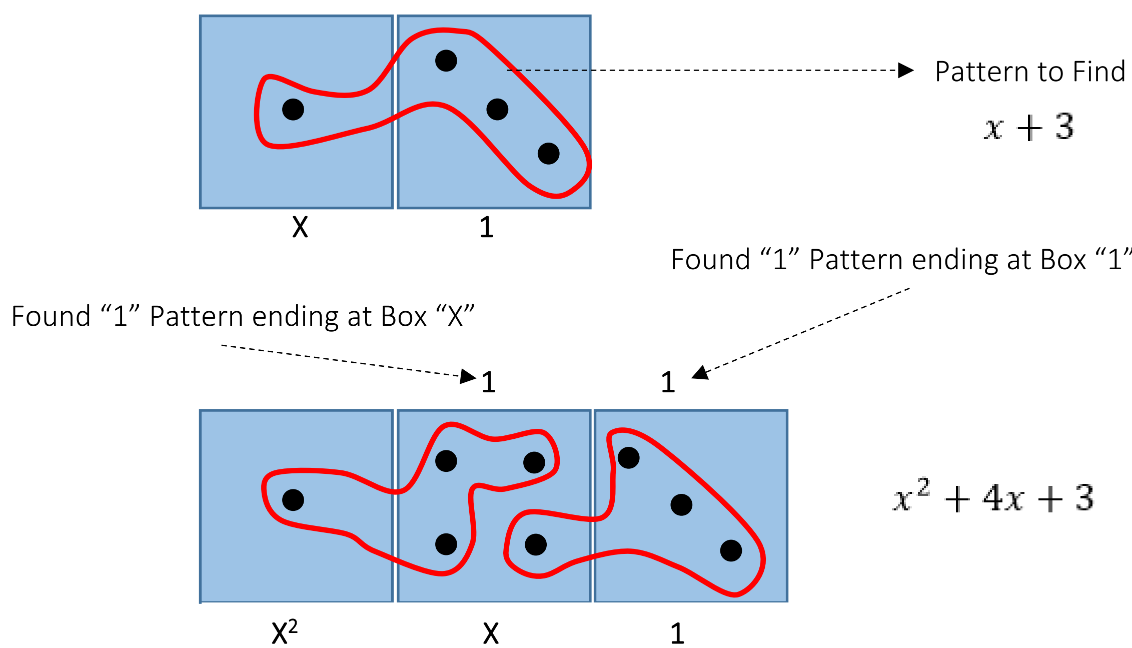
[Author of "**Mathematical Approach to Puzzle Solving**"]

James Tanton's Exploding Dots is a revolutionary idea that help students understand and connect the various math concepts, and learn them in a simple, unified and fun way. Visit http://gdaymath.com/courses/exploding-dots/ to discover this amazing math machine.

In this article I'm going to explain how Exploding Dots can be used to solve complex polynomial division involving two variables, in a simple way.

Just to recap, here is how we use Exploding Dots for polynomial division involving one variable.

$$(\boldsymbol{x^2+4x+3})\ divided\ by\ (\boldsymbol{x+3})$$



So the answer is (1 * X) + (1 * 1) = $x+1$

FIGURE 1: Polynomial Division Involving One Variable

Wow, it can't get simpler and easier than this!! Even a 3rd grade student can now solve an 8th grade polynomial division problem with great ease and fun.

Now let us switch our attention to polynomials involving two variables. For illustration, let us take a simple problem. Find the quotient when $(a^3b^2+2a^2b+a)$ is divided by (a^2b+a).

This is where the two dimensional Exploding Dots come into picture. So we have one Exploding Dots machine along the horizontal direction, and one along the vertical direction.

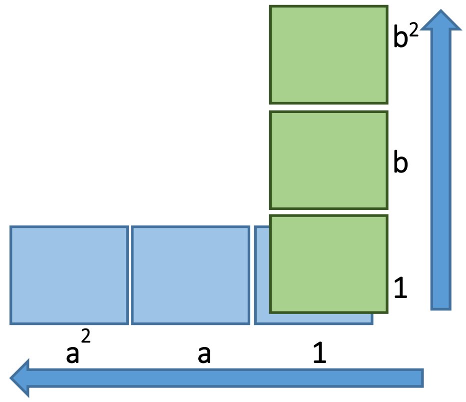


FIGURE 2: Exploding Dots in Two Dimensions

Let us now see how this fabulous machine helps us solve the above polynomial division problem.

Step 1: To begin with, find out the highest degree of each variable in the dividend polynomial. Let it be "m" and "n". Construct a rectangular grid of dimension "m+1" x "n+1".

In our example, for the expression $(a^3b^2 + 2a^2b + a)$, we need to construct a grid of dimension 4 x 3, since "m" (the highest degree of variable "a") is 3, and "n" (the highest degree of variable "b") is 2.

Step 2: Mark the dots and anti-dots, based on the co-efficient of each term, within the corresponding box of the rectangular grid. Identifying the correct box is very similar to the way we mark the position of a "shot" in the "Battleship" game.

In our example, for the expression$(a^3b^2 + 2a^2b + a)$, we mark one dot in the box at position (a^3, b^2), two dots in the box at position (a^2, b) and one dot in the box at position (a, 1).

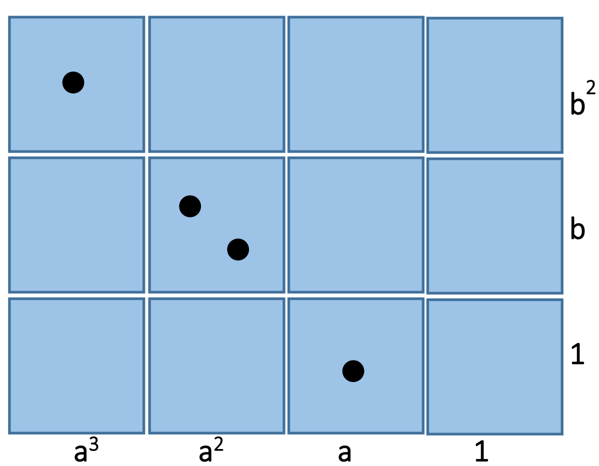


FIGURE 3: $(a^3b^2 + 2a^2b + a)$ represented using Exploding Dots in Two Dimensions

Step 3: Repeat the same process for the divisor polynomial, which in our case is $(a^2b + a)$.

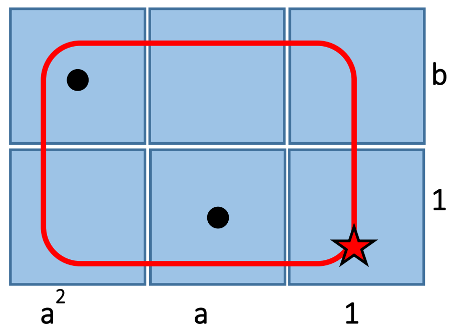


FIGURE 4: $(a^2b + a)$ represented using Exploding Dots in Two Dimensions

Step 4: The red colored loop in **Figure 4** represents the pattern to find. This pattern needs to be found in the grid in **Figure 3**. The loop has a red star at the **bottom right** corner. This indicates the pattern start/end position. The loop has to start from this point, go through all the boxes, enclosing the dots and return back to the same point. For each pattern we find in **Figure 3**, we need to record the position of the red star, to derive the answer.

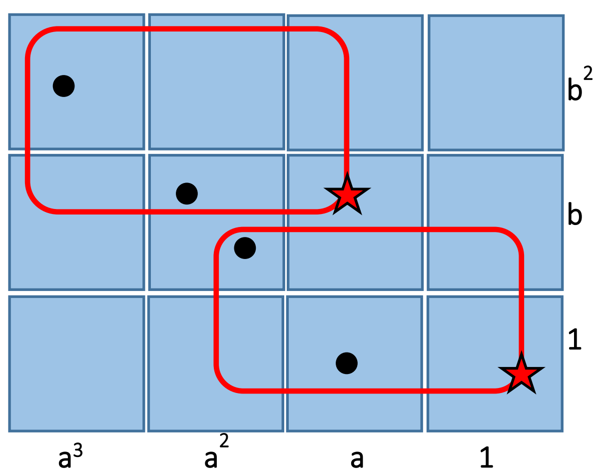


FIGURE 5: Finding the pattern $(a^2b + a)$

Step 5: We found one pattern with the red star in the box at position (a, b). And we found one more pattern with the red star in the box at position (1, 1). So the answer for this polynomial division problem is (1 * (a * b)) + (1 * (1 * 1)) = $(\boldsymbol{ab} + \mathbf{1})$

Thus $(\boldsymbol{a^3b^2} + \mathbf{2}\boldsymbol{a^2b} + \boldsymbol{a})$ divided by $(\boldsymbol{a^2b} + \boldsymbol{a})$ results in the quotient $(\boldsymbol{ab} + \mathbf{1})$

Wow, isn't this exciting? All the "Battleship Game Experts" can now call themselves as "Polynomial Division Experts". Thanks to Exploding Dots!!

Let us look at couple of more examples. Exploding Dots is so self-explanatory, I will just draw the boxes and mark the patterns for these examples.

Example 1: $(4a + a^2b^2 - 2ab^2 + 2ab^3 - 2a^2 - 4ab + b^2 - 2)$ divided by $(a^2 - 2a + 2ab + 1)$

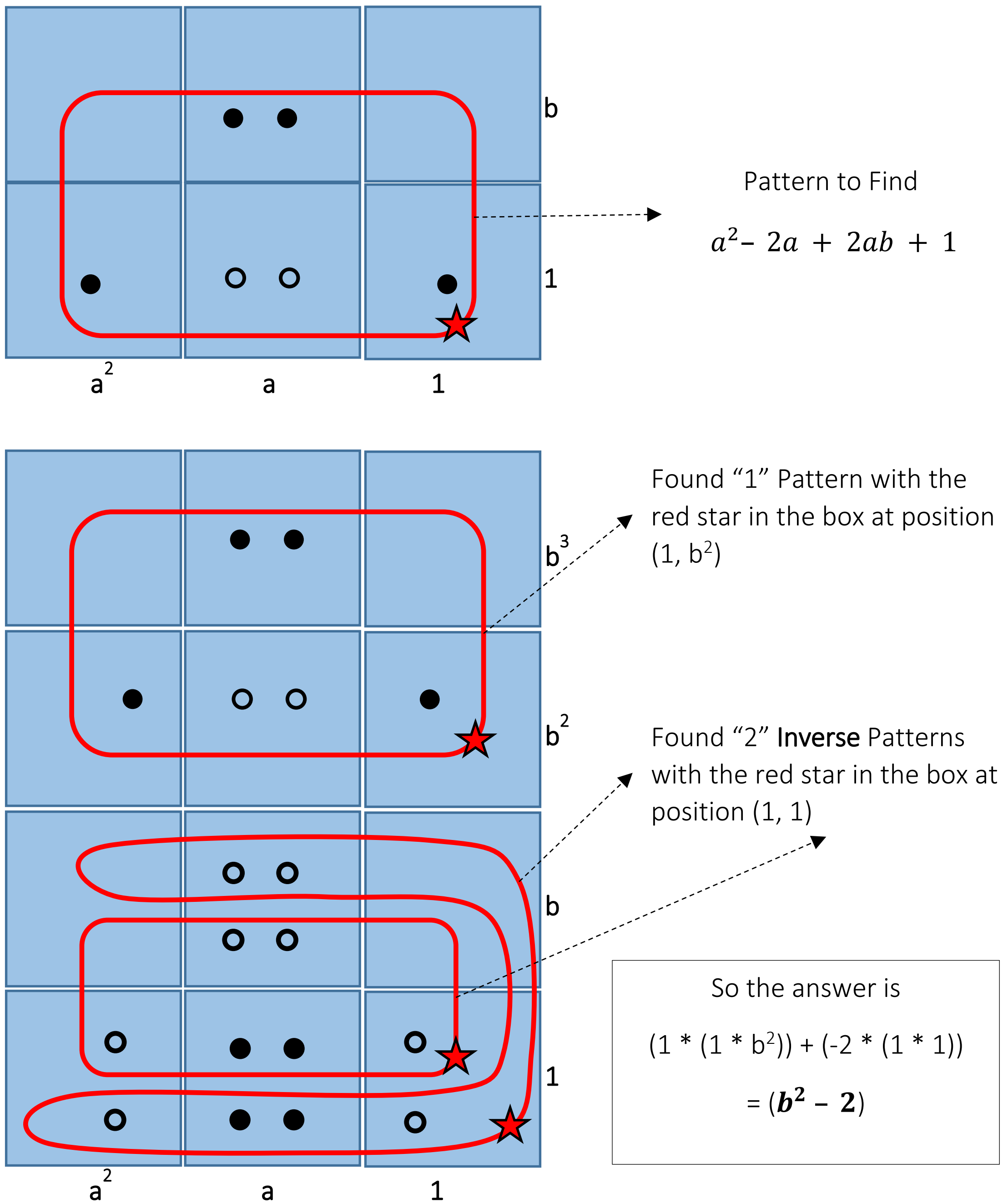


FIGURE 6: An Example illustrating the power of Exploding Dots

Did you believe this could be so easy when you first saw the problem?

Example 2: $(x^3 - y^3)$ divided by $(x - y)$

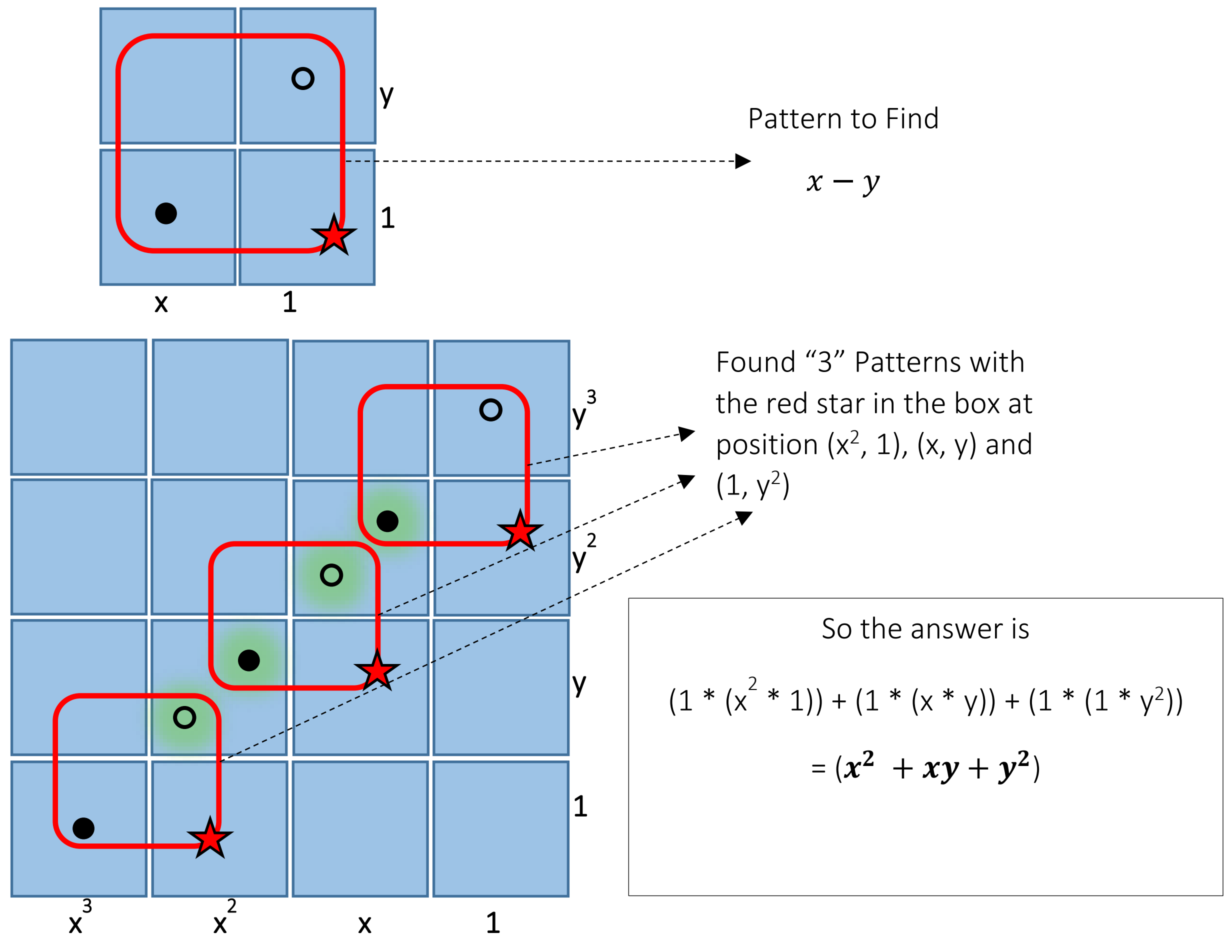


FIGURE 7: Yet another example illustrating the power of Exploding Dots

Wasn’t this example equally fascinating? Just can’t resist solving all the polynomial division problems from the text book! So what are you waiting for? Go grab your mathematics text book, and start having fun.

Now how about a polynomial involving 3 variables? Ummh…that’s even more interesting. Consider a simple problem – $(a^2bc + ab)$ divided by $(ac + 1)$. And think about cubes and cuboids. Yes, Exploding Dots in three dimensions. Let's reserve that for another day. Goodbye and Have Fun.