# INTERLEAVED EXPLODING DOTS

# AND THE

# ROMAN NUMERALS

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James Tanton's Exploding Dots is a revolutionary idea that help students understand and connect the various math concepts, and learn them in a simple, unified and fun way. Visit http://gdaymath.com/courses/exploding-dots/ to discover this amazing math machine.

In this article I'm going to explain how Exploding Dots can be used to understand the Roman Numeral System in an intuitive way. I'm also going to demonstrate how this intuitive technique helps us in performing arithmetic operations easily with Roman numbers, and also in converting Hindu-Arabic Numbers into Roman Numbers in an incredibly easy fashion.

Consider a simple [1 ← 10] Exploding Dots Machine with the initial box value as one, as shown below.

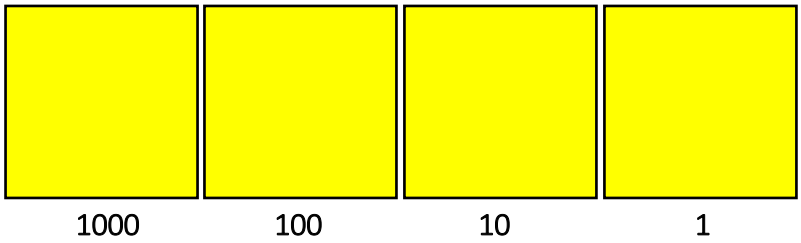



FIGURE 1: [1 ← 10] Exploding Dots Machine with the initial box value as 1

Consider another [1 ← 10] Exploding Dots Machine with the initial box value as five, as shown below.

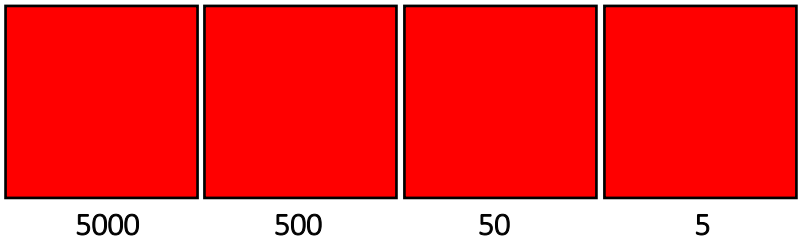



FIGURE 2: [1 ← 10] Exploding Dots Machine with the initial box value as 5

Let us now combine these two machines by interleaving the boxes to get a Special Interleaved Exploding Dots Machine as shown below.

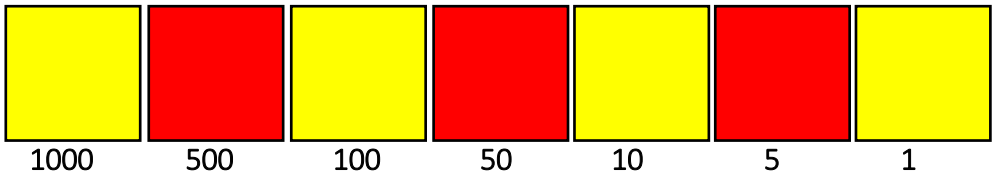



FIGURE 3: A Special Interleaved Exploding Dots Machine

Because we have interleaved two different [1 ← 10] Exploding Dots Machine, the combined machine is no longer a [1 ← 10] Exploding Dots Machine. This special machine has the following rules for explosion of dots.

Rule #1: Four dots in the Yellow Box explode to form one dot in the Red Box on its immediate left, and one anti-dot in the same Yellow Box. So this is a special variant of a [1 ← 5] machine.

Rule #2: Two dots in the Red Box explode to form one dot in the Yellow Box on its immediate left. This is a simple [1 ← 2] machine.

Thus by interleaving two [1 ← 10] machine together, we got a machine that acts as a [1 ← 5] machine and [1 ← 2] machine alternately. Let us look at how this machine works by taking a very simple example. Let us see how we can represent "6" in this machine.

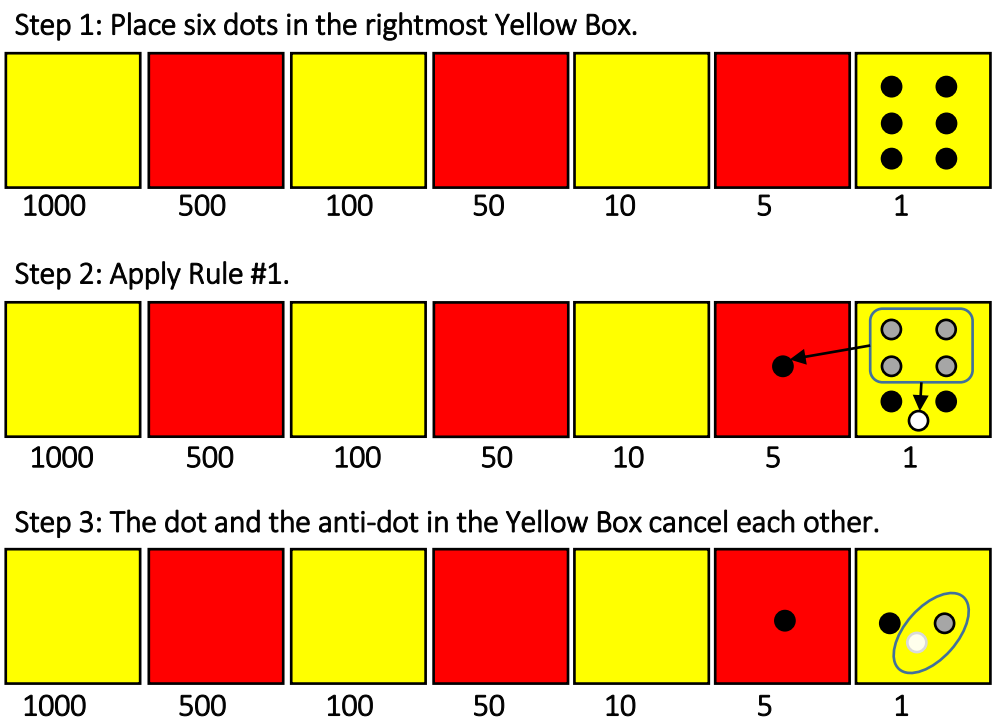



FIGURE 4: Representation of "6" in this Special Interleaved Exploding Dots Machine

Thus “6” is represented by a dot in the Red Box labelled “5” and a dot in the Yellow Box labelled “1”. Now let us replace the labels 1, 5, 10, 50, etc. with some very familiar symbols as shown below.

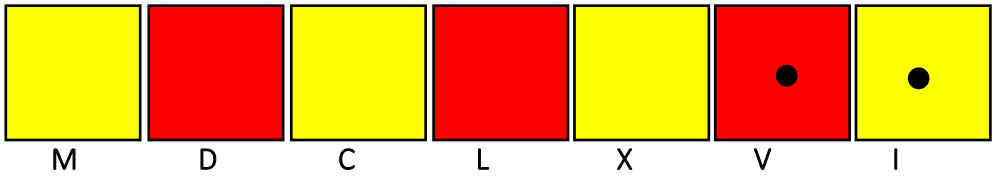



FIGURE 5: The Roman Numeral Exploding Dots Machine

Voila!! Ladies and Gentlemen, introducing to you, the “**Roman Numeral Exploding Dots Machine**”.

It is no wonder that “6” = “VI” in Roman Numerals.

Are you excited? I’m sure you are! Let us explore further.

Considering the constraints in Roman Numerals, any valid Roman Number represented using this machine adhere to the following rules.

**Rule A**: The anti-dots can be present **only** in the **Yellow Boxes** labelled “I”, “X” and “C”.

**Rule B**: For every anti-dot there should be a matching dot in any one of the two boxes to its immediate left. This anti-dot and dot pair is called a “**Link**”. The dots and anti-dots in a link don’t participate in explosions. There are two types of link as shown below.

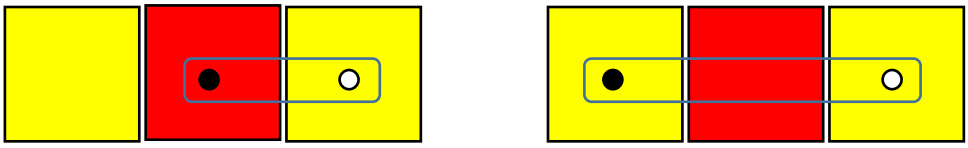


FIGURE 6: Two Types of Links

**Rule C**: Links cannot overlap each other, but can be grouped together to form a chain as shown below. So the right end of the chain lies in a Yellow Box, while the left end of the chain can lie in a Red Box or a Yellow Box.

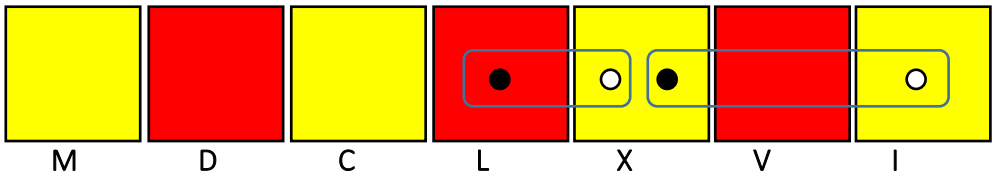



FIGURE 7: A Chain consisting of two Links - Link “LX” and Link “XI”

**Rule D**: Boxes containing links cannot have any free dots except for the Yellow Box that holds the left end of a chain (left end of the leftmost link of a chain). The Yellow Box (that holds the left end

of a chain) can contain up to a maximum of 3 free dots. So as illustrated in the below figure, the Yellow Box labelled “X” cannot have any free dots, while the Yellow Box labelled “C” can.

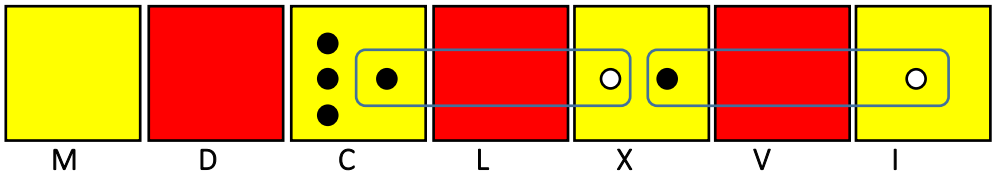



FIGURE 8: The left end of the chain is in the Yellow Box labelled “C”

So given a Roman Number, we can easily validate if it is a valid Roman Number or not in an easy visual way using the following steps.

**Step #1**: Starting from the left, for each Roman Number symbol, represent it as

(a) A dot in the box labelled with that symbol, if the symbol on its right is of the same or lesser value.
OR
(b) An anti-dot in the box labelled with that symbol, if the symbol on its right is of a higher value.

**Step #2**: Check if the Roman Numeral Exploding Dots notation adheres to all the 4 rules above. If so, it is a valid Roman Number. Otherwise it is an invalid Roman Number.

Let us work out an example.

Is “MDCCLIXVI” a valid Roman Number?

**Step #1**: Represent it in a Roman Numeral Exploding Dots machine.

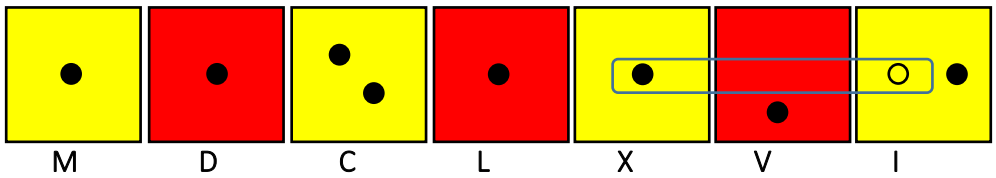



FIGURE 9: Representation of “MDCCLIXVI” using the Exploding Dots Machine.

**Step #2**: Applying the rules, we can clearly see that “Rule D” is violated here in the Yellow Box labelled “I” and the Red Box labelled “V”.

Thus “MDCCLIXVI” is not a valid Roman Number.

Now let us look at another type of problem. Given a number in Hindu-Arabic form, convert it into a Roman Number. Again, there exists an easy way to do this. But before we get into that, we need to familiarize ourselves on how to represent the numbers 1 to 9 using Roman Numeral Exploding Dots Machine.

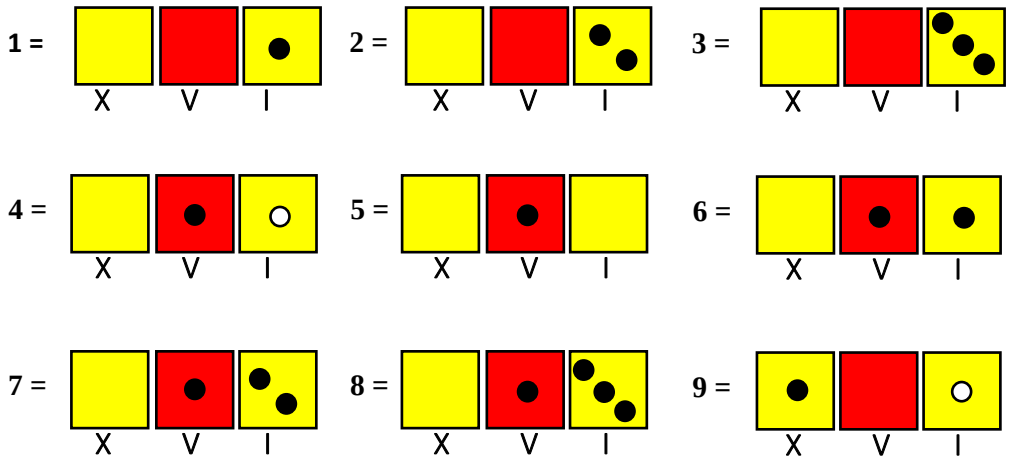



FIGURE 10: Representation of 1 to 9 in Roman Numeral Exploding Dots Machine

Once you have memorized the pattern for each of these nine numbers, it's time to get started with an example problem. Represent "949" in Roman Number form.

Step #1: Layout the Roman Numeral Exploding Dots Machine as shown in the below diagram.

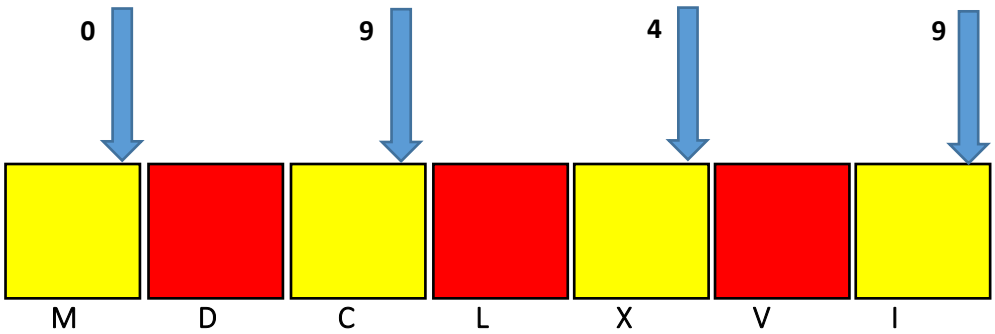



FIGURE 11: Positioning of the digits in the Roman Numeral Exploding Dots Machine

Note: The Units digit of the Hindu-Arabic Number corresponds to the box labelled "I". The Tens digit of the Hindu-Arabic Number corresponds to the box labelled "X". The Hundreds digit of the Hindu-Arabic Number corresponds to the box labelled "C". And the Thousands digit of the Hindu-Arabic Number corresponds to the box labelled "M".

**Step #2**: Insert the pattern for each digit at the corresponding position in the Exploding Dots Machine.

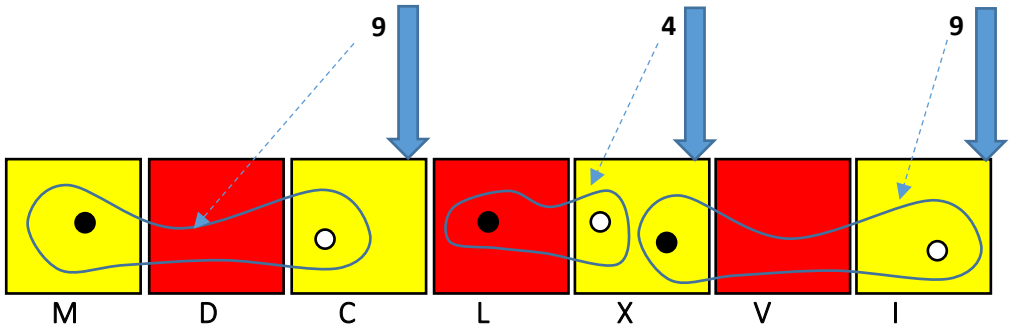



**FIGURE 12**: Pattern insertion for each digit in the Roman Numeral Exploding Dots Machine

**Step #3**: Starting from the left, for each dot, write the symbol of the box in which the dot is present. And for each anti-dot, write the symbol of the box in which the anti-dot is present, & underline it.


**FIGURE 13**: Deriving the initial Roman Number representation

**Step #4**: Rewrite all the "underlined" symbols by jumping one position to the left to get the final answer.

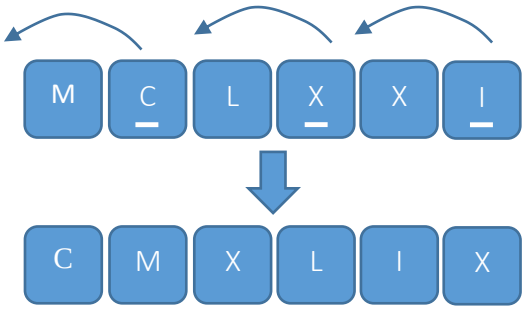


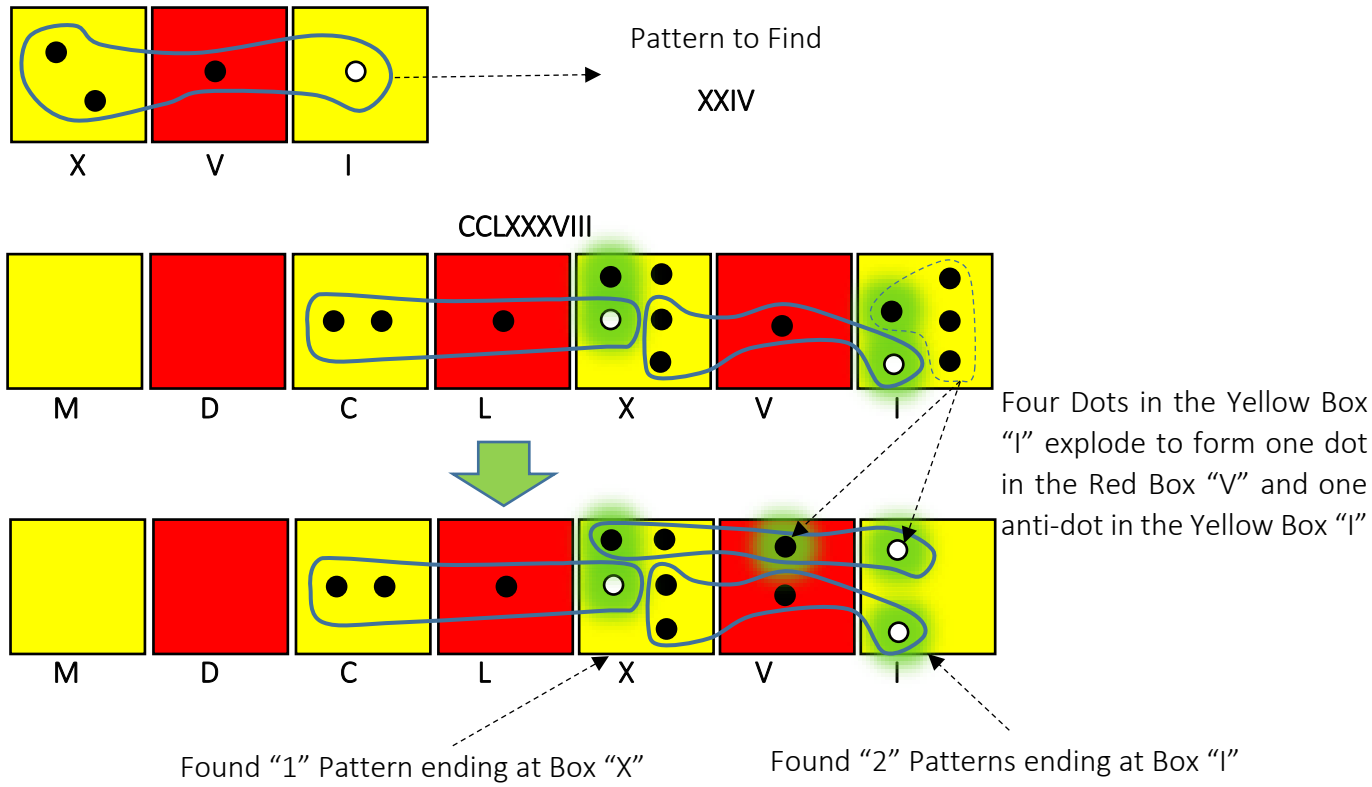

**FIGURE 14**: Deriving the final Roman Number representation

So "949" is nothing but "**CMXLIX**" in Roman Number form. Wow, that's an easy and error free way to convert Hindu-Arabic numbers into Roman Number Form.

Now coming to the last leg of this exciting journey, let us talk about division of Roman Numbers. How can we find the answer for "**CCLXXXVIII**" divided by "**XXIV**"?

One way is to convert them into Hindu-Arabic numbers, perform the division, and re-convert the result back into Roman Number form. But can we divide the Roman Numbers without converting them back and forth?

Yes, we can! And Exploding Dots helps us do that in a super easy way.




So the answer for "**CCLXXXVIII**" divided by "**XXIV**" is "**XII**"

**FIGURE 15**: Division of Roman Numbers using Roman Numeral Exploding Dots Machine

Presto!! This is as easy as it can ever get. Exploding Dots not only makes it easy but also more fun.

Before we end our adventurous journey, a quick note on the pattern matching in case of Roman Numeral Exploding Dots Machine. While matching the pattern, we need to also ensure that the color of the boxes matches too.

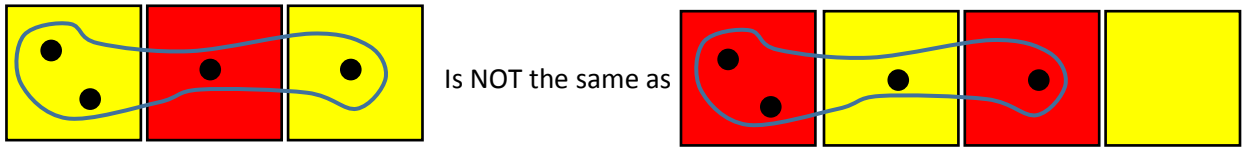



**FIGURE 16**: Pattern Matching needs to consider the color of the boxes too

So we have come to the end of a fabulous adventure with Roman Numbers using the revolutionary Exploding Dots concept. Or should I say that the adventure has just begin?