PROGRESSIONS AND METRIC CONVERSION

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Of the many number sequences we come across in our daily lives, progressions are the most common. Let us have some fun with one such simple number sequence called the **Geometric Progression**.

Geometric progression is a number sequence in which each term is formed by multiplying the previous term with a fixed non-zero number. This fixed number is called the "**Ratio**", since it is the quotient of the current term divided by the previous term. So what would be starting number? The answer is simple. You can use any number to start off with. Let us create our own geometric progression.

Starting with 5, and a ratio of "10", our geometric progression would be 5, 50, 500, 5000 and so on.

5, 50, 500, 5000, 50000, ...

Starting Number - 05
Ratio - 10

FIGURE 1: A Simple Geometric Progression

Okay, so that was indeed very simple. Now what is the use of Geometric Progressions? Where do we find them in real life? Why do we need to study about them? These are some important questions we need to answer while starting to learn anything for that matter. In this article we will look into one particularly astonishing occurrence and usage of geometric progressions.

Our first encounter with Geometric Progressions dates back to the time we learnt about place value in the decimal number system (probably in the kindergarten). The ones place value, the tens place value, the hundreds place value and so on, written one after the other actually forms a geometric progression.

1, 10, 100, 1000, 10000, ...

Starting Number - 01
Ratio - 10

FIGURE 2: Place Values in Decimal Number System form a Geometric Progression

Take any number system such as binary, ternary or octal. The place values in these number systems would also form a geometric progression. And the geometric progression "Ratio" would be the "**radix**" or the "**base**" of that number system.

$$1, 2, 4, 8, 16, 32, 64, \ldots$$

Starting Number - 01
Ratio - 02

FIGURE 3: Place Values in Binary Number System form a Geometric Progression

So far, so good. Now let us see how we use the geometric progressions unknowingly when we convert metrics from one unit to another.

How many decades is 230 years? Super-duper easy. The answer is "23" decades. What is actually happening behind the scenes is that we are shifting the place value sequence to the left by one place, and re-evaluating the number as shown below.

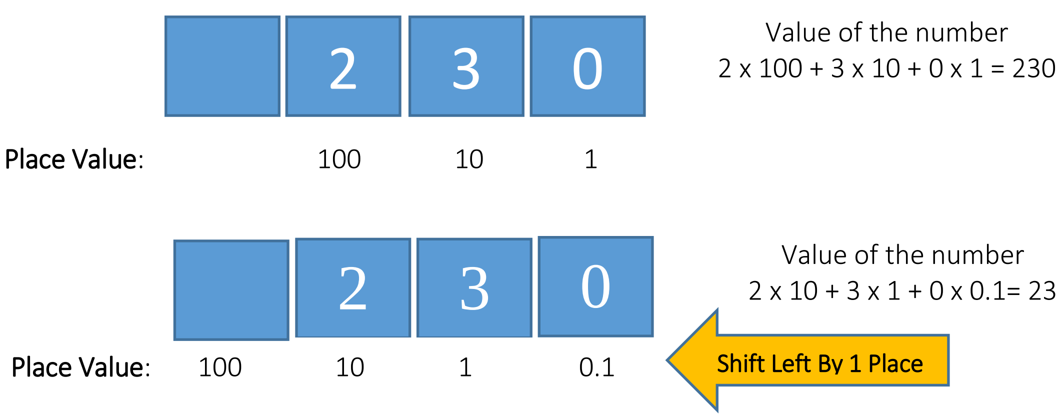

FIGURE 4: Converting years into decades

We can do this only because the conversion factor is 10, i.e. 10 years make a decade. And also "10" is one of the numbers in the geometric progression formed by the place value sequence in the decimal number system. (Refer **Figure 2**). We have to shift "Left" by only one place because "10" is just one position away from the ones place.

Quick Question: Can you figure out when we would need to shift "Right" instead of "Left"?

Let us take another example. How many meters is 3450 centimeters? Simple again, "34.5" meters. It is exactly the same thing happening behind the scenes. This time the conversion factor is 100, i.e. 100 centimeters make one meter. Again "100" is one of the numbers in the geometric progression formed by the place value sequence in the decimal number system. (Refer **Figure 2**). We have to shift "Left" by two places because "100" is two positions away from the ones place.

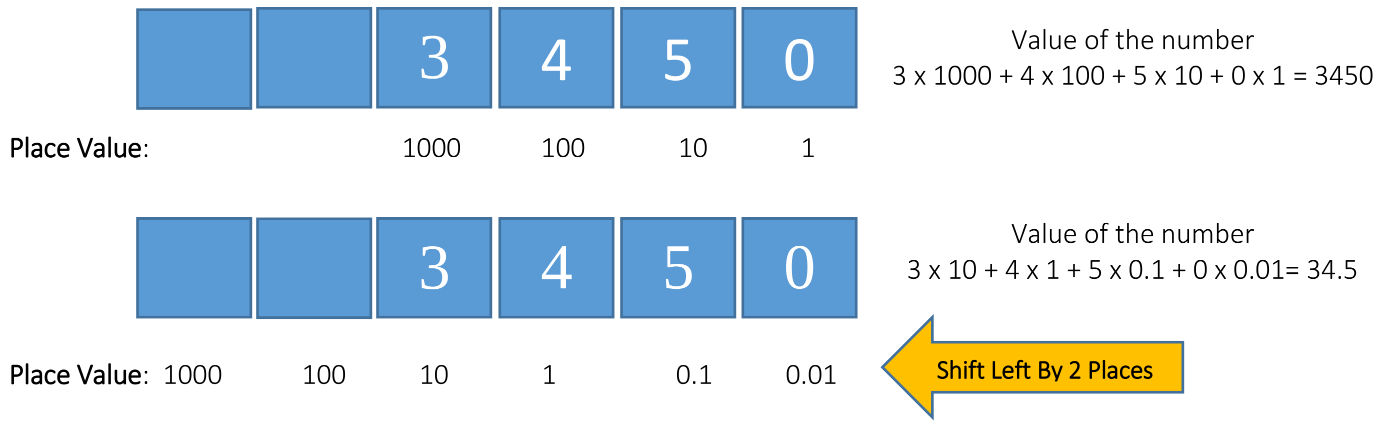

FIGURE 5: Converting centimeters into meters

Because we are so mechanically used to do these kind of simple conversions at the flick of a second, we don't bother to understand what's going on behind the scenes – the dancing and hopping of the geometric progressions.

Once we understand this, it becomes very easy to extend this mechanism to other metric conversions where the conversion factor is not a multiple of 10. Let us take the example of converting Pounds to Kilograms. The conversion factor here is 2.2, i.e. 2.2 pounds make 1 kilogram.

Can you think of a similar mechanism where we can convert pounds into kilograms by mere shifting? Let's explore. Let us create our own geometric progression where the starting number is 1, and the "ratio" 2.2

1, 2.2, 4.84, 10.65, 23.43, ...

Starting Number - 01
Ratio - 2.2

FIGURE 6: Geometric Progression formed by Place Values in a Base 2.2 Number System

Welcome to the fractional base number system! This sequence represents the place values in base 2.2 number system. To keep the sequence simple, we will round off the numbers. Because we are rounding off the numbers, the ratio is approximately 2.2. And we get a simple sequence as below.

1, 2, 5, 11, 23, ...

Starting Number - 01
Ratio - 2.2 (approximately)

FIGURE 7: A Beautiful Sequence based on Base 2.2 Number System

This sequence is not a pure geometric progression, but quite close enough to the geometric progression whose starting number is 1, and the ratio "2.2". Let us use this beautiful sequence as the place value sequence to represent the numbers. The following table shows how we can represent the numbers 1 to 10 using this new number system.

Number in decimal	Representation using new number system
1	001 (0x5 + 0x2 + 1x1 = 1)
2	010 (0x5 + 1x2 + 0x1 = 2)
3	011 (0x5 + 1x2 + 1x1 = 3)
4	020 (0x5 + 2x2 + 0x1 = 4)
5	100 (1x5 + 0x2 + 0x1 = 5)
6	101 (1x5 + 0x2 + 1x1 = 6)
7	110 (1x5 + 1x2 + 0x1 = 7)
8	111 (1x5 + 1x2 + 1x1 = 8)
9	120 (1x5 + 2x2 + 0x1 = 9)
10	121 (1x5 + 2x2 + 1x1 = 10)

FIGURE 8: Represent the numbers 1-10 in the new number system

Now back to the original challenge of coming up with a mechanism to convert pounds into kilograms by mere shifting. Consider an object that weighs 29 pounds. In this new number system, 29 would be represented as 10101. Applying the same principles of shifting, we see that 29 pounds equals 13 kilograms.

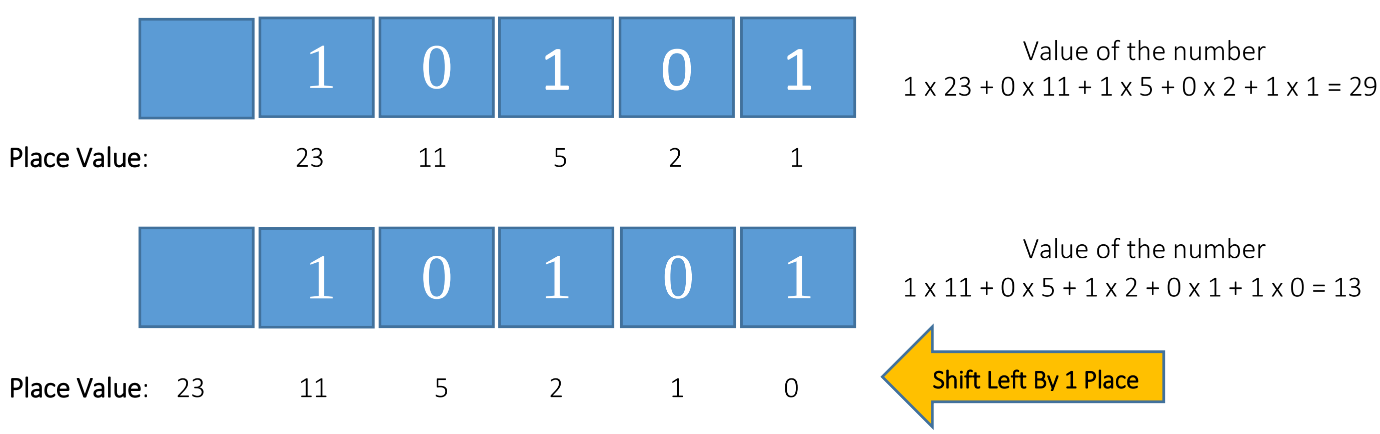

FIGURE 9: Converting pounds into kilograms

Wow, wasn't that wonderful? If we represent the value of pounds in this new number system, we can use the same shifting mechanism to convert pounds into kilograms.

Note: 29 pounds actually equals 13.15 kilograms. Because we are rounding off the numbers in the original geometric progression, we got a sequence which was not a pure geometric progression, but quite close to the original geometric progression. Hence we got the answer that 29 pounds equals 13 kilograms which is not accurate, but quite close to the accurate value. (And good enough for casual use in day-to-day life)

One final piece to finish off with. Let us look at converting kilometers into miles. The conversion factor is 1.6, i.e. 1.6 kilometers make one mile. I would leave it as an exercise for the reader to come up with the mechanism of shifting, for converting kilometers into miles. A quick illustration below should give you the clue.

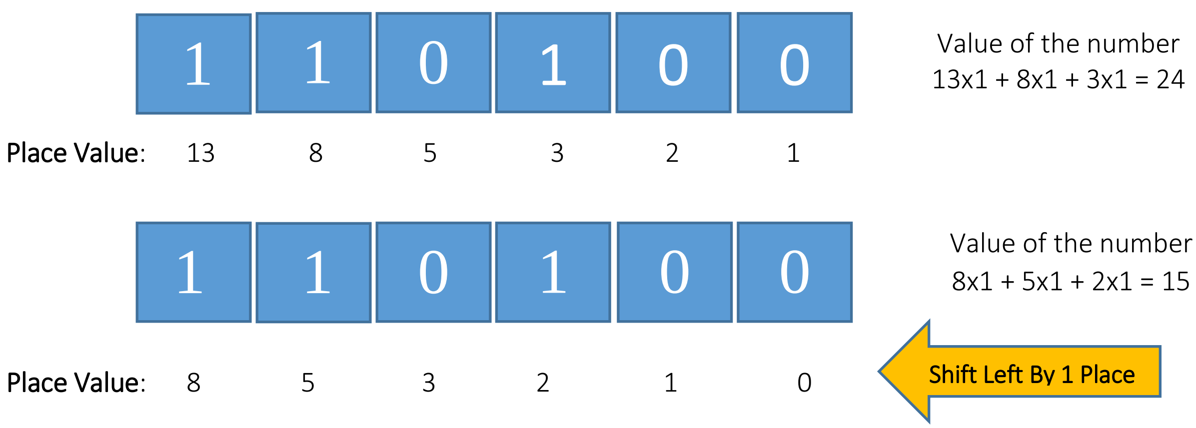

FIGURE 10: 24 kilometers is nothing but 15 miles

Do you recognize the place value sequence 1, 2, 3, 5, 8, 13, …? Yes, the famous **Fibonacci Sequence**. We have just encountered the Fibonacci Number System. Voila, no wonder that 1.6 is the value of "**phi**", the **Golden Ratio**.

What a fantastic application we have seen the Fibonacci series come into use! If you are super excited, then try coming up with a similar mechanism for converting kilometers into nautical miles. Here the conversion factor is 1.85, i.e. 1.85 kilometers make one nautical mile. Do you see some amazing number sequence blooming out?

Try experimenting with all different metric conversions, and discover the amazing geometric progressions and number sequences that come out.